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ON THE RADIAL LIMITATION OF THE SOLAR MAGNETIC FIELD

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ABSTRACT. The Drift current, Dynamo and Diamagnetic theories of the radial limitation of the general magnetic field of the sun are discussed in the light of the present knowledge of Physics. In the first article a summary is given of the mathematical results, obtained in a previous paper of the authoress, for the behaviour of an electron in electric, magnetic and gravitational fields. In the second article the results are applied to analyse the drift current and dynamo theories of the radial limitation in a solar atmosphere of isothermal equilibrium under gravity. In the third article the diamagnetic theory of this limitation has been developed, the diamagnetism being now induced, not by the classical method of Schrödinger, but by the quantisation in the motion of the electrons in the magnetic field. It is concluded that according to none of these theories the magnetic field can be radially limited to the extent as found from the observations of Hale in 1913. Similar conclusions seem to follow from a study of the polarisation of non-uniform rotation of the sun in its magnetic field.

INTRODUCTION

One of the greatest triumphs of the application of the spectroscopic methods to Astrophysics is the discovery of the magnetic field of the sun. The study of the polarisation of the doublets of the absorption lines in sun-spots and its comparison with that of Zeeman effect led Hale,[1] in 1908, to the discovery of strong magnetic field of the sun-spots, the intensity of which in the case of large spots amounts to even as large as 4000 Gauss. The discovery gave a stimulus for the search of a general magnetic field of the sun and thanks to the untiring perseverance of Hale and his collaborators in Mount Wilson Observatory, who at last announced in 1913 that the sun possessed a general magnetic field much like that of the earth with the following principal properties:

(*i*) The magnetic axis of the sun, like that of the earth, was found to be not coincident with the axis of rotation but situated at a distance of 4° from it and revolving round it in a period of 31.5 days.

(*ii*) The magnetic field, though similar in some respects to that of a uniformly magnetised sphere, showed distinct deviations from it in the distribution of intensity with respect of latitude. Taking the sun to be a uniformly magnetised sphere, the polar intensity of the field has been estimated from the observations as derived from two different latitude zones ($-10°$ to $+10°$) and $\pm$ ($10°$ to $45°$) and the estimates are found to be very different, that from the equatorial regions being 1.84 times as great as that from higher latitude zones.

(*iii*) The magnetic field intensity was found to decrease rapidly outwards in the radial direction falling from 150 Gauss (after the correction of Rosseland) to an amount (perhaps 30 Gauss) too small to be measured between the base and top of the reversing layer. The decrease was estimated to take place in a height of about 300 Km.

Different theories have been put forward by Chapman [2] and Gunn [3] to explain the cause of the radial limitation of solar magnetic field. The methods of both are based on a consideration of the motions of the charged particles that exist in the solar atmosphere under the action of magnetic, electric and gravitational fields of the sun. In general, when a charged particle on the surface of the sun is subject to a horizontal and south to north magnetic field, the vertically downward gravitational field and the vertical electric (upwards for ions and downwards for electrons according to Pannekoek-Rosselands' Theory) field of the sun, as is really the case at the equator, it would describe a cycloidal path in a plane at right angles to the meridian plane together with its motion along the horizontal magnetic field. The motion may therefore be resolved into two parts: a translational motion which is in a direction perpendicular to both the fields and motion in a helix with its axis of spiralling along the magnetic field of the sun. The first part is known as drift current and has been considered in details by Chapman. He shows that, as a result of this drift, the charges of opposite signs separate out in opposite directions resulting in a net eastward current in the solar atmosphere which reduces the magnetic field rapidly outwards along the radius. According to Gunn, however, the limitation is brought about by the second part of the motion, namely the spiralling of the electrons round the magnetic field, which, he showed, would produce diamagnetism in the solar atmosphere and reduce the field in the required manner. Gunn's method is based on an idea which has been used by Schrödinger to explain the diamagnetism in metals. The electrons in a metal under the action of a magnetie field describe circular arcs in their free paths due to Lorentz force and this produces diamagnetism, as, according to classical electrodynamics, the electrons describing circles are equivalent to magnetic doublets whose axes are in a direction opposed to the applied magnetic force.

Though the results of Chapman and Gunn agreed fairly well with the observations of Hale, it was shown later on by Cowling [4] in two consecutive papers that such an agreement was really spurious. He showed that none of these explanations was tenable in an atmosphere of thermodynamic equilibrium, the effects of Chapman and Gunn when properly considered being cancelled out. Because of this failure Ferraro [5] made a fresh investigation of formulating the 'Dynamo' theory of the radial limitation. In this theory the eastward current, necessary for the limitation, is produced by the motion of the Sun's atmosphere in meridian planes across the radial component of the magnetic field, the motion being, as suggested by Bjerknes, from the poles to the equator with a back poleward current beneath. The theory was already considered by Chapman but the effect produced by this way was shown by him to be quite negligible. But Ferraro's investigation showed that an appreciable limitation of the field could be obtained if the mean velocity of the atmosphere along the meridian corresponded to the motion of sun-spots from latitude 45° to the equator in 11 years. It was supposed that such a motion of the solar atmosphere might result from the unequal heating of the poles and the equator but a calculation of Eddington[6] showed that velocity for this cause was far too small to be of importance.

Thus we find ourselves faced with a dilemma. The failure of the attempts to explain the radial limitation of the general magnetic field of the sun tends us to suspect the genuineness of the observational evidence for its existence.[7] Before we can definitely question about the existence of such a limitation it is desirable that a fresh theoretical investigation along the line of present developments in physics should be made to clarify the present position. It should be mentioned that there are some obscurities in Cowling's method of calculation. Further we know today that the diamagnetic effect considered by Gunn after Schrödinger's method is really spurious. It has been shown by Landau[8] and Darwin[9] that under the action of a magnetic field the motion of the electrons is no more continuous but descrete, or quantised as it is called and the change in energy induced by this quantisation is responsible for the observed diamagnetism in metals. The whole problem therefore requires a thorough revision on a more rigorous mathematical basis incorporating into it the new outlook about the behaviour of the electrons in a magnetic field. This will be attempted in the present paper. In the first part we shall summarise the mathematical results for the behaviour of an electron in electric, magnetic and gravitational fields. In the second part we discuss the results of Chapman in the light of our present theory. In the third part we develope the diamagnetic theory as the cause of the radial limitation, the diamagnetism being induced, as mentioned above, by the quantisation of the motion of the electrons in the magnetic field.

EXPRESSION FOR THE CURRENT DENSITY[10]

The current density, $\vec{I}$, *i.e.*, the total charge passing through unit area in unit time is defined by

$$\vec{I} = 2e\iiint \vec{v} f_1 d\kappa_x d\kappa_y d\kappa \qquad \ldots (1)$$

where f_1, the change in the distribution function of the electron due to resultant action of the electric and magnetic fields on the one hand and the interaction between electrons and ions on the other hand is given by

$$f = f_0 + f_1 \qquad \dots (2)$$

f and f_0 being the Fermi distribution functions in the presence and absence of the fields respectively.

f_1 is determined from the usual Maxwell-Boltzmann's equation

$$\frac{\partial f}{\partial t} + \frac{h}{m}\left(\vec{\kappa} \text{ grad}_{\vec{r}} f\right) + \frac{1}{h}\left(\vec{F} \text{ grad}_{\vec{k}} f\right) = -\frac{f_1}{\tau} \qquad \dots (3)$$

$\vec{F}$ being the total Lorentz force acting on the electron.

Thus
$$\vec{F} = \vec{X} + \frac{e}{c}\left[\vec{v}\ \vec{H}\right] \qquad \dots (4)$$

where the first term $\vec{X}$ includes all the forces, electrical, electrostatic, gravitational, etc., and the second one gives the force due to the magnetic field $\vec{H}$. $\vec{\kappa}$ is the wave vector, $\vec{v}$ is the velocity of the electron and τ is the time of relaxation.

Solving equation (3) for f_1 and substituting its value so obtained in (1) we obtain after some simplifications

$$\vec{I} = K\vec{V} + L\left[\vec{V}\ \vec{\omega}_L\right] + M\,\vec{\omega}_L\left(\vec{\omega}_L\ \vec{V}\right) \qquad \dots (5)$$

where
$$K = -\frac{2eh^2}{3m^2}\iiint \frac{\tau \frac{\partial f_0}{\partial \epsilon}\kappa^2 d\kappa_x d\kappa_y d\kappa_z}{1+\tau^2\omega_L^2} \qquad \dots (6)$$

$$L = -\frac{2eh^2}{3m^2}\iiint \frac{\tau^2 \frac{\partial f_0}{\partial \epsilon}\kappa^2 d\kappa_x d\kappa_y d\kappa_z}{1+\tau^2\omega_L^2} \qquad \dots (7)$$

$$M = -\frac{2eh^2}{3m^2}\iiint \frac{\tau^3 \frac{\partial f_0}{\partial \epsilon}\kappa^2 d\kappa_x d\kappa_y d\kappa_z}{1+\tau^2\omega_L^2} \qquad \dots (8)$$

$$\vec{V} = \vec{X} - \frac{kT}{A}\text{ grad}_{\vec{r}} A \qquad \dots (9)$$

$$A = \frac{nh^3}{2(2\pi mkT)^{\frac{3}{2}}}, \quad \omega_L = \frac{eH}{mc} \qquad \dots (10)$$

n being the number of electrons per unit volume.

Before we discuss the theories of the limitation some remarks regarding the approximations to be used in our mathematical calculations for the evaluation of the integrals occurring in the current expression are necessary. It is to be particularly noted that the integrals (6), (7) and (8) can only be evaluated in the two limiting cases, namely when the electrons are or are not free to spiral between two collisions,

i.e., (*i*) $\tau\omega_L >> 1$, $H >> H_0$ ... (11)

or (*ii*) $\tau\omega_L << 1$, $H << H_0$, ... (12)

H_0 being a limiting magnetic field defined by

$$H_0 = \frac{mc}{c\bar{\tau}}, \qquad \ldots (13)$$

where

$$\bar{\tau} = 1{,}89 \frac{T^{\frac{3}{2}}}{z^2 n^+ \bar{J}} \qquad \ldots (14)$$

$$\bar{J} = \log (t+1) - \frac{t}{t+1} \qquad \ldots (15)$$

$$t = 3{\cdot}3 \times 10^{13} \frac{T^2}{n^+} \qquad \ldots (16)$$

and z = number of free electrons per atom, n^+ = number of ions per unit volume.

We have calculated in the following table the values of H_0 at the base and the top of the reversing layer. It shows that the spiralling of the electrons has already begun to be prominent even at the base of the reversing layer. We shall therefore use the approximation (11) in our present calculations. We have also added in the table the conductivities σ_1 and σ_2 ; σ_1 is the conductivity in the radial direction and σ_2 the conductivity in a direction at right angles to the meridian plane, which we call the transverse conductivity.

n^+ in c.c.	T in °K	H in Gauss	H_0 in Gauss	σ_1 in E.S.U.	σ_2 in E.S.U.
5×10^{13}	5500	150	56·6	$6{\cdot}1 \times 10^{12}$	$4{\cdot}8 \times 10^{12}$
$2{\cdot}5 \times 10^{12}$	5500	30	3·4	$4{\cdot}6 \times 10^{11}$	$1{\cdot}2 \times 10^{12}$

The first set of values corresponds roughly to conditions at the base of the reversing layer, whereas the second set gives the values at a height of 300 km. We have assumed thereby that the gas is singly ionised, *i.e.*, $z=1$.

DRIFT CURRENT AND DYNAMO THEORIES OF RADIAL LIMITATION

Drift Current Theory: We specialise the problem. We take the current layer at the equator. The gas is assumed to be in isothermal equilibrium under gravity, the density changing exponentially with the height. If X-axis be taken vertically upwards and the Z-axis horizontal in the south-north direction, the Y-axis will be horizontal and eastward. The electric and magnetic fields will vary along the X-axis. We obtain the current in Y-direction, *i.e.*, the drift current as

$$\mathrm{I}_y = -\mathrm{L}\left(\mathrm{X} - \frac{k\mathrm{T}}{\mathrm{A}}\frac{\partial \mathrm{A}}{\partial x}\right)_{\omega_i} \qquad \ldots \quad (17)$$

or we obtain

$$\mathrm{I}_y = -\frac{\mathrm{L}e\mathrm{H}}{mc}\left(\mathrm{X} - \frac{k\mathrm{T}}{n}\frac{\partial n}{\partial x}\right) \qquad \ldots \quad (18)$$

where L is given by (7).

In X we must include, as mentioned before, gravitational as well as a vertical eloctrostatic field which is set up by the tendency of the light electrons to spread out farther in the vertical direction than the heavier ions; the drift due to electric field, as is evident from (18) and (7), being the same for the ion and the electron, no electric current will be produced and hence we shall leave this out of account from our present discussion. The electrostatic field developed on the sun's surface has been discussed by Pannekoek, Rosseland and Milne; it is of the order of 4370 Volts. The electrostatic field will be such that

$$e\mathrm{E} = \tfrac{1}{2}(m_i - m_e)g. \qquad \ldots \quad (19)$$

This is upward on the positive ion, and downward on the electron; hence

$$\mathrm{X} = -m_e g - \tfrac{1}{2}(m_i - m_e)g = -\tfrac{1}{2}(m_i + m_e)g. \qquad \ldots \quad (20)$$

The same force will also be acting on the ion.

(I) Now in the case of homogeneous atmosphere in which density is constant the expression (18) is reduced to

$$\mathrm{I}_y = \frac{\mathrm{L}e\mathrm{H}}{2mc}(m_i + m_e)g \qquad \ldots \quad (21)$$

and further taking the time of relaxation to be constant, we obtain from (21) and (7), by putting

$$\tau = \frac{l}{v}, \quad \mathrm{R} = \frac{mvc}{e\mathrm{H}} \qquad \ldots \quad (22)$$

where l = mean free path and R is the radius of the spiralling,

$$\mathrm{I}_y = \frac{l^2}{l^2 + \mathrm{R}^2} \cdot \frac{nc}{2\mathrm{H}}(m_i + m_e)g \qquad \ldots \quad (23)$$

which is exactly the Chapman's expression.

(II) But when the atmosphere is not homogeneous, we have

$$n^{+}=n=n_0 e^{-\frac{(m_i+m_e)}{2kT}gx} \qquad \dots \quad (24)$$

where n_0 is the number of particles at the base of the reversing layer.

Thus from (18), (20) and (24) we obtain $I_y=0$, that is, there is no current in the eastward direction. Thus we see clearly that Chapman's drift current in eastward direction is completely masked by westward current produced by the inhomogeneity of the gas. Drift current theory therefore fails to give any limitation of the solar magnetic field.

Dynamo Theory : In Dynamo theory, as already mentioned, the requisite eastward current is produced by the motion of the gas in the solar atmosphere from pole to the equator in the meridian plane across the radial component (though very small, indeed) of the sun's magnetic field. The eastward current due to induced electromotive force is therefore given by

$$i=\sigma_2 \frac{v}{c} H_r = \frac{\sigma_2 v}{c} \cdot \frac{r_0}{r} H_\theta \cot\theta \qquad \dots \quad (25)$$

where we have taken after Ferraro[5]

$$H_r = \frac{r_0}{r} H_\theta \cot\theta \qquad \dots \quad (26)$$

which is correct at least in order of magnitude and where θ is the colatitude of the region, r_0 is given by (29), v is the velocity of the gas in the meridian plane, H_r, H_θ are the radial and horizontal components of the field and σ_2 is the transverse conductivity.

Now since

$$i=-\frac{c}{4\pi}\frac{\partial H_\theta}{\partial r}, \qquad \dots \quad (27)$$

we obtain by neglecting the variation of r compared with a, the radius of the sun,

$$H_\theta = H_0 e^{-\frac{r}{r_0}}, \qquad \dots \quad (28)$$

where

$$r_0=\left(\frac{ac^2 \tan\theta}{4\pi\sigma_2 v}\right)^{\frac{1}{2}}. \qquad \dots \quad (29)$$

We now take $\theta \sim 45°$ and assume that the motion of the solar atmosphere in the meridian planes to be due to the unequal heating of the sun at the poles and the equator owing to the solar rotation. In this case the velocity v comes out of the order of 10^{-3} cm./sec. as shown by Eddington. Substituting these values of θ

and v and taking σ_2 from the table given before we find that the Dynamo theory becomes quite inadequate to explain the required limitation as observed.

REVISED DIAMAGNETIC THEORY OF RADIAL LIMITATION

Following the methods of Landau, Darwin, Peirls and others it can be proved that the motion of the electrons of an ionised atmosphere in the presence of a magnetic field becomes quantised and it is this quantisation which renders the medium diamagnetic, that is, equivalent to a distribution of intensity of magnetisation directed opposite to the original magnetic field at each point. As this quantisation effect is entirely a quantum mechanical phenomenon having no analogy in classical physics and the deduction of the formula for the intensity of magnetisation is fundamentally different from that of Schrödinger and Gunn, we prefer to give first a short derivation of the formula. The quantisation is due to the fact that the circular motion of the electron in a plane at right angles to the direction of the impressed magnetic field can be resolved into two harmonic motions vibrating at right angles, which can take up only descrete energy values in quantum mechanics.

Let the magnetic field H be parallel to the Z-axis. Then the motion of the electron will be composed of two parts—one linear along the Z-axis and the other circular, hence quantised, in the plane XY. The energy will, therefore, be given by

$$\epsilon = \epsilon_{trans} + \epsilon_{long} = (2n+1)\mu H + \frac{h^2 \kappa_z^2}{2m}, \quad \ldots \quad (30)$$

where μ is the Bohr magneton $= \dfrac{eh}{4\pi mc}$ ... (31)

and n the quantum number.

To find the magnetism we must first calculate the energy of the system. Following the usual method of statistical mechanics, we have

$$N = \frac{2V}{h^3} \iiint f dp_x dp_y dp_z = 2V \iiint f d\kappa_x d\kappa_y d\kappa_z \quad \ldots \quad (32)$$

with the transformations $\kappa_x = \kappa \cos\alpha$, $\kappa_y = \kappa \sin\alpha$,

we have
$$N = 4\pi V \iint f \kappa d\kappa d\kappa_z . \quad \ldots \quad (33)$$

Now
$$\int \kappa d\kappa = \tfrac{1}{2}(\kappa_{n+1}^2 - \kappa_n^2) = \frac{2m\mu H}{h^2} . \quad \ldots \quad (34)$$

Thus $$N=\frac{8\pi V m\mu H}{h^2}\sum_n\int f d\kappa_z$$

$$=\frac{8\pi V m\mu H}{h^2}A_H\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}e^{-\frac{h^2}{2mkT}\kappa_z^2}d\kappa_z\,.\,e^{-\frac{(2n+1)\mu H}{kT}} \quad \ldots \quad (35)$$

or, since $$\sum_{n=0}^{\infty}e^{-(2n+1)y}=\frac{1}{2\sinh y}\,,$$

we obtain by integration

$$N=\frac{2V}{h^3}\frac{\mu H}{kT}(2\pi mkT)^{\frac{3}{2}}.\frac{A_H}{\sinh\frac{\mu H}{kT}}\,. \quad \ldots \quad (36)$$

Or, since $$\frac{\mu H}{kT}<<1,$$

$$N=\frac{2V}{h^3}\cdot\frac{\mu H}{kT}(2\pi mkT)^{\frac{3}{2}}A_H\cdot\frac{kT}{\mu H}\left[1-\frac{1}{6}\left(\frac{\mu H}{kT}\right)^2\ldots\ldots\right]. \quad \ldots \quad (37)$$

Or, introducing A, the value of A_H without the magnetic field, which is equal to

$$\frac{nh^3}{2(2\pi mkT)^{\frac{3}{2}}}\,,$$

we obtain $$A_H=A\left[1+\frac{1}{6}\left(\frac{\mu H}{kT}\right)^2\right]. \quad \ldots \quad (38)$$

The energy is now given by

$$E=\frac{8\pi V m\mu H}{h^2}\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}f\epsilon_n d\kappa_z$$

$$=\frac{8\pi V m\mu H}{h^2}A_H\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}\left\{(2n+1)\mu H+\frac{h^2}{2m}\kappa_z^2\right\}e^{-\left(\frac{h^2}{2mkT}\kappa_z^2+\frac{(2n+1)\mu H}{kT}\right)}d\kappa_z$$

or since $$\sum_{n=0}^{\infty}(2n+1)\mu H e^{-\frac{(2n+1)\mu H}{kT}}=\mu H\cdot\frac{\cosh\frac{\mu H}{kT}}{2\sinh^2\frac{\mu H}{kT}}\,,$$

$$E=\frac{2V}{h^3}\frac{\mu H}{kT}.(2\pi mkT)^{\frac{3}{2}}.\frac{A_H}{\sinh\frac{\mu H}{kT}}\left\{\mu H\coth\frac{\mu H}{kT}+\frac{kT}{2}\right\}. \quad \ldots \quad (39)$$

Expanding $\coth \frac{\mu H}{kT}$ and $\sinh \frac{\mu H}{kT}$ and substituting the value of A_H we obtain after some simplifications,

$$E = N\left(\tfrac{3}{2}kT + \frac{\mu^2 H^2}{3kT} + \ldots\right). \qquad \ldots (40)$$

For $H = 0$, average energy is thus reduced to $E = \frac{3}{2}kT$.

The intensity of magnetisation due to the quantisation is, therefore, given by

$$I = -\frac{\partial E}{\partial H} = -\frac{2}{3}\cdot\frac{n\mu^2 H}{kT}. \qquad \ldots (41)$$

Now, since at the equatorial regions where we chiefly confine our attention, H is approximately horizontal, the layer is everywhere tangentially magnetised and the potential at a point $P(R, \theta, \phi)$ due to the magnetised shell of radius r' will be given by

$$\Omega_p = dr' \iint I_{\theta'} \frac{\partial}{r'\partial\theta'}\left(\frac{1}{r}\right) r'^2 \sin\theta' d\theta' d\phi', \qquad \ldots (42)$$

where r is the distance between P and any point $Q\ (r', \theta', \phi')$ on the shell. Thus

$$r^2 = R^2 + r'^2 - 2Rr' \cos\lambda, \qquad \ldots (43)$$

where $\cos\lambda = \cos\theta \cos\theta' + \sin\theta \sin\theta' \cos(\phi - \phi')$.

Integrating partially we have from (42)

$$\Omega_p = -dr' \iint \frac{1}{r}\left[\frac{1}{r'\sin\theta'}\frac{\partial}{\partial\theta'}(I_{\theta'}\sin\theta')\right] r'^2 \sin\theta' d\theta' d\phi'$$

$$= dr' \iint \frac{\sigma}{r} r'^2 \sin\theta' d\theta' d\phi \qquad \ldots (44)$$

where $$\sigma = -\frac{1}{r'\sin\theta'}\frac{\partial}{\partial\theta'}(I_{\theta'}\sin\theta'). \qquad \ldots (45)$$

The potential at P is thus equivalent to that due to a distribution of magnetic matter of surface density σ given by (45).

Since σ is a function of θ' only and not ϕ' it can be expanded in a series of zonal Harmonics.

Thus $$\sigma = \sum_n \sigma_n P_n(\cos\theta'), \qquad \ldots (46)$$

where σ_n is the coefficient of the zonal Harmonic of order n and given by

$$\sigma_n = \frac{2n+1}{2}\int_{-1}^{+1} P_n(\mu)\sigma(\mu)d\mu. \qquad \ldots (47)$$

Also
$$\frac{1}{r} = \frac{1}{\sqrt{R^2 + r'^2 - 2Rr' \cos \lambda}}$$
$$= \sum_m \frac{R^m}{r'^{m+1}} P_m(\cos \lambda), \qquad \dots (48)$$

R being less than r', as P is an internal point.

$$\text{Thus,}\quad \Omega_p = dr' \sum_n \sum_m \sigma_n \cdot \frac{R^m}{r'^{m-1}} \iint P_n(\mu) P_m(\cos \lambda)\, d\mu d\phi' \qquad [\mu = \cos \theta']$$

$$= 4\pi dr' \sum_n \frac{\sigma_n}{2n+1} \cdot \frac{R^n}{r'^{n-1}} P_n(\cos \theta). \qquad \dots (49)$$

The magnetic force F at P, which is equal to $\frac{1}{R}\frac{d\Omega}{d\theta}$ is thus given by

$$F = 4\pi dr' \sum_n \frac{\sigma_n}{2n+1} \frac{R^{n-1}}{r'^{n-1}} \frac{dP_n(\cos \theta)}{d\theta}. \qquad \dots (50)$$

As we are considering the regions very near to the spherical shell, R will be approximately equal to r' ; so that with the help of (47) we obtain

$$F = 2\pi dr' \sum_n \frac{dP_n(\cos \theta)}{d\theta} \int_{-1}^{+1} P_n(\mu) \sigma(\mu) d\mu . \qquad \dots (51)$$

Now in our special case when the value of σ is given by (45) with $I_{\theta'}$ taken from (41) the above expression becomes much simplified and we obtain at once

$$F = -\frac{16\pi}{9} \cdot \frac{n\mu^2}{kT} H_\theta \frac{dr'}{r'} , \qquad \dots (52)$$

where $H_\theta = H_e \sin \theta$, H_e being the value of the magnetic field at the equator so that the change dH in H due to a layer of thickness dr' is

$$dH_\theta = -\frac{16\pi}{9} \frac{n\mu^2}{kT} H_\theta \frac{dr'}{r'} . \qquad \dots (53)$$

Since we may neglect the variation of r' compared with a, the radius of the sun, it follows from (53), when we remember

$$n = n_0 e^{-\frac{Mgr'}{kT}} , \qquad \dots (54)$$

that
$$\log \frac{H}{H_0} = -\frac{16\pi}{9} \cdot \frac{n_0 \mu^2}{aMg} \left(1 - e^{-\frac{Mgr}{kT}} \right) . \qquad \dots (55)$$

Here M is the average mass of a particle in an atmosphere of Russel Mixture.

$$\text{When} \qquad \frac{Mgr}{kT} \ll 1, \qquad H = H_0 e^{-\frac{r}{r_0}} \qquad \ldots (56)$$

$$\text{and when} \qquad \frac{Mgr}{kT} \gg 1, \qquad H = H_0 e^{-\frac{kT}{Mgr_0}} \qquad \ldots (57)$$

where H_0 is the value of H at the base of the reversing layer and

$$r_0 = \frac{9kTa}{16\pi n_0 \mu^2} \cdot \qquad \ldots (58)$$

We, therefore, find that according to the Diamagnetic Theory, the Magnetic field first decreases exponentially with the height reaching a constant value when the height is large compared with $\frac{kT}{Mg} \backsim 100$ km. Though this result seems to be in accordance with the observations, a little calculation shows that the value of r_0 is so large as to produce no decrease at all of the magnetic field in the sun's atmosphere.

CONCLUSION

We are, therefore, led to conclude from the above considerations that none of these theories can be a proper explanation of the radial limitation of the solar magnetic field as observed by Hale in 1913. There is no *a priori* reason also why the Sun's field will have such a limitation when there is no such thing in the case of the Earth's magnetic field except that it decreases with height according to Schmidt's formula. Similar conclusions have also been reached by Ferraro[11] from a study of the polarisation of non-uniform rotation of the sun in its magnetic field. He has shown that for the polarisation to be set up the angular velocity of the sun should be constant over the surfaces traced out by revolving the magnetic lines of force around the magnetic axis and which would mean no limitation of the radial field. It may be noted in this connection that the recent investigation of Millikan and Neher[12] on the latitude distribution of cosmic ray intensity and its theoretical analysis by Epstein[13] show that the sun is surrounded by a magnetic field whose intensity at the polar region should be about 25 Gauss. It will be interesting if the cosmic ray investigations in this line can furnish an evidence for the existence of this radial limitation.

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REFERENCES

1 G. E. Hale, *Mount Wilson Contributions*, Nos. 50, 61, 71, 72, 148.

2 S. Chapman, *M. N.*, **89**, 59 (1928).

3 R. Gunn, *Phys. Rev.*, **33**, 614 (1929).

4 T. G. Cowling, *M. N.*, **90**, 140 (1929) ; *ibid*, **92**, 407 (1932).

5 V. C. A. Ferraro, *M. N.*, **95**, 280 (1935).

6 A. S. Eddington, *M. N.*, **90**, 54 (1929).

7 V. C. A. Ferraro, *Observatory*, **61**, 241 (1938).

8 L. Landau, *Zs. f. Phys.*, **64**, 629 (1930).

9 C. G. Darwin, *Proc. Camb. Phil. Soc.*, **27** (1931).

10 B. Majumdar, *Tras. Bose. Research. Inst.*, **14**, 17 (1938-1939).

11 V. C. A. Ferraro, *M. N.*, **97**, 458 (1937).

12 I. S. Bowen, R. A. Millikan and H. V. Neher, *Phys. Rev.*, **53**, 855 (1938).

13 P. S. Epstein, *Phys. Rev*, **53**, 862 (1938).